Int. J. Heat Mass Transfer. Vol. 30, No. 6, pp. 1159–1168, 1987
Printed in Great Britain

0017-9310/87 \$3.00+0.00

The theory of advanced multi-layer thin film heat transfer gauges

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(*Received* 4 *October* 1985 *and in final form* 28 *May* 1986)

Abstract—This paper describes the theory of a transient method of measuring heat transfer rate to metal substrates coated with an electrical insulator, using thin film resistance thermometers. This builds on the already well-established system which uses semi-infinite insulating substrates. It is intended that the new technique will have application in rotating turbine test rigs, since there is at present a lack of suitable instrumentation which can be easily manufactured, and which does not interfere with the flow. The new system described here shows that multi-layer substrate gauges can be used. This paper presents analyses of layered gauges and gives sample predictions and calibrations.

1. INTRODUCTION

THE USE of thin film resistance thermometers on insulating substrates for measuring heat transfer rates to turbine blades in short duration transient cascade facilities is well documented [1, 2]. Turbine blades are machined from machinable glass ceramic (Corning Macor) and the surface is readily instrumented with thin film resistance thermometers. The depth to which the heat penetrates into the insulator is small, so that the substrate may be considered semi-infinite. Electrical analogues [3], are used to obtain the heat transfer rate from the surface temperature signal. The use of machinable glass is, for structural reasons, limited to stationary cascade facilities. There is however, a necessity for instrumentation which can be utilized on metal turbine blades in fully rotating turbine test rigs, since there is a major gap between the data provided by static cascade testing (which does not fully model the situation of the rotor) and the rotating systems, where the difficulties of instrumentation have severely restricted the quality of measurements.

This paper describes the theory behind the use of thin film resistance thermometers on multi-layered substrates. This work is described in more detail in ref. [9] and the practical applications are to be published [10].

2. SEMI-INFINITE SUBSTRATES—TYPE 1 GAUGE

Thin film gauges which operate on the semi-infinite principle (Fig. 1), have been well described [1]. A small constant current is passed through the film and the changes of voltage across this thermometer are proportional to the changes in surface temperature T_s. The governing equation (assuming that the effect of the surface sensor is negligible) is the unsteady heat conduction equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

where α is the thermal diffusivity, with the boundary condition

$$\dot{q}(x=0) = \dot{q}_s = -k\frac{\partial T}{\partial x} \quad \text{on} \quad x = 0$$

which is solved to give the Laplace transform of the surface heat transfer rate

$$\bar{\dot{q}}_s = (\rho c k s)^{1/2}\bar{T}_s.$$

It is possible to compute $\dot{q}$ from a digitally recorded T signal, but the numerical error when digitizing the film voltage waveform gives rise to noise on the reconstructed $\dot{q}$ signal [4], and so it is more usual to use a lumped resistance capacitance transmission line as an electrical analogue (Fig. 2) to convert the film voltage into a current proportional to $\dot{q}$ and to record this current. For a continuous RC transmission line, the governing equations are

$$\frac{\partial i}{\partial x} = -c\frac{\partial v}{\partial t}$$

$$i = \frac{1}{r}\frac{\partial v}{\partial x}$$

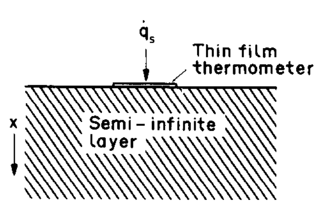

FIG. 1. Thin film gauge on semi-infinite layer.

NOMENCLATURE

c	specific heat capacity, capacitance/unit length	T	temperature
$h(t)$	gauge step calibration function	T_s	surface temperature
i	current	t	time
I_0	constant current through film	v_a	analogue output voltage
k	thermal conductivity	v	voltage
$\dot{q}$	heat transfer rate	v_0	initial film voltage
$\dot{q}_s$	surface heat transfer rate	x	distance.
Q	constant heat transfer rate		
r	resistance per unit length		
s	Laplace transform variable		
$\bar{}$	Laplace transform		

Greek symbols

α	temperature coefficient of resistance, diffusivity
ρ	density.

$$\frac{\partial^2 v}{\partial x^2} = rc\frac{\partial v}{\partial t}$$

$$\bar{i} = \left(\frac{c}{r}\right)^{1/2} s^{1/2}\bar{v}$$

from which it can be seen that i is analogous to $\dot{q}$ and V and T.

3. TWO-LAYER GAUGES

3.1. *Semi-infinite backwall—type 2 gauge*

The gauge has two layers (Fig. 3)

$0 \leqslant x \leqslant a$ electrically insulating layer $i = 1$

$a \leqslant x \leqslant \infty$ metal substrate $i = 2$.

The governing equations are

$$\frac{\partial^2 T_i}{\partial x^2} = \frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} \quad \text{for} \quad i = 1, 2 \tag{2}$$

with the boundary conditions

$$-k_1\frac{\partial T_1}{\partial x} = \dot{q}_s \qquad \text{on} \quad x = 0$$

$$T_1 = T_2 \qquad \text{on} \quad x = a$$

$$k_1\frac{\partial T_1}{\partial x} = k_2\frac{\partial T_2}{\partial x} \qquad \text{on} \quad x = a$$

$$\frac{\partial T_2}{\partial x} = 0 \qquad \text{on} \quad x = \infty.$$

Taking Laplace transforms and solving for $\bar{T}_1$, $\bar{T}_2$ gives

$$\bar{T}_1 = \frac{\bar{\dot{q}}_s}{k_1}(\alpha_1/s)^{1/2}\left(\frac{(1+\sigma)\exp\{-(x-a)(s/\alpha_1)^{1/2}\}+(1-\sigma)\exp\{(x-a)(s/\alpha_1)^{1/2}\}}{(1+\sigma)\exp\{a(s/\alpha_1)^{1/2}\}-(1-\sigma)\exp\{-a(s/\alpha_1)^{1/2}\}}\right) \tag{3}$$

$$\bar{T}_2 = \frac{2}{k_1}\bar{\dot{q}}_s(\alpha_1/s)^{1/2}\left(\frac{\exp\{(a-x)(s/\alpha_2)^{1/2}\}}{(1+\sigma)\exp\{a(s/\alpha_1)^{1/2}\}-(1-\sigma)\exp\{-a(s/\alpha_1)^{1/2}\}}\right) \tag{4}$$

For this model, the metal substrate is considered to be semi-infinite, and the thermal capacity of the surface sensor is again considered to be negligible.

where

$$\sigma = \left(\frac{\rho_2 c_2 k_2}{\rho_1 c_1 k_1}\right)^{1/2}.$$

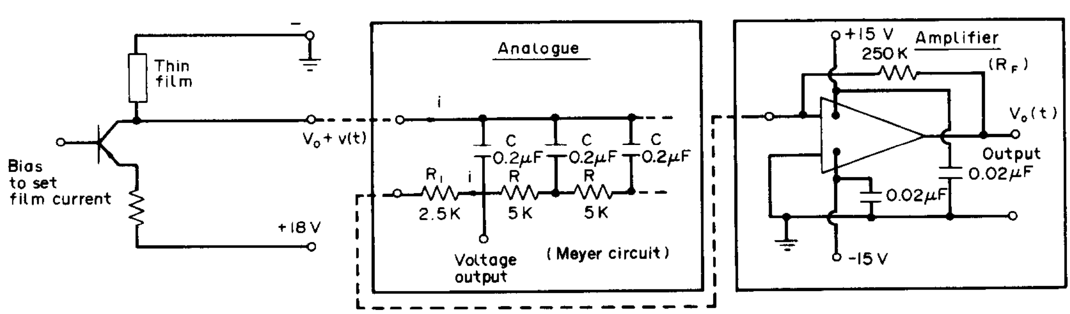

FIG. 2. Electrical analogue circuit for obtaining heat transfer rate from measured surface temperature.

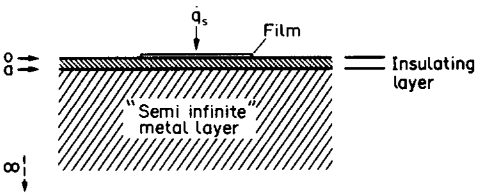

FIG. 3. Thin film gauge on two-layered composite comprising electrically insulating layer on semi-infinite metal.

If the heat flux is assumed to be constant, Q (a step input at $t = 0$), then

$$\bar{\dot{q}}_s = \frac{Q}{s}.$$

Inverting equation (3) and putting $x = 0$, gives the surface temperature

$$T_1(x=0,t) = T_s$$

$$T_s = \frac{2Q}{(\rho_1 c_1 k_1)^{1/2}}\left((t/\pi)^{1/2} + 2\sum_{n=1}^{\infty} A^n \left\{(t/\pi)^{1/2} \times \exp\left(\frac{-n^2a^2}{\alpha_1 t}\right) - \frac{na}{\alpha_1^{1/2}}\operatorname{erfc}\left(na/(\alpha_1 t)^{1/2}\right)\right\}\right) \quad (5)$$

where

$$A = \frac{1-\sigma}{1+\sigma}.$$

This can be used later to predict the surface temperature.

3.2. *Finite backwall model—type 3 gauge*

If the metal substrate is finite then the model has two-layers (Fig. 4)

$$-a \leqslant x \leqslant 0 \quad \text{insulator} \quad i = 1$$

$$0 \leqslant x \leqslant b \quad \text{metal substrate} \quad i = 2.$$

The equations to be solved are

$$\frac{\partial^2 T_i}{\partial x^2} = \frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} \quad \text{for} \quad i = 1,2$$

with the boundary conditions

$$-k_1\frac{\partial T_1}{\partial x} = \dot{q}_s \quad \text{on} \quad x = -a$$

$$T_1 = T_2 \quad \text{on} \quad x = 0$$

$$k_1\frac{\partial T_1}{\partial x} = k_2\frac{\partial T_2}{\partial x} \quad \text{on} \quad x = 0$$

$$\frac{\partial T_2}{\partial x} = 0 \quad \text{on} \quad x = b.$$

Taking the Laplace transforms of the above equations gives

$$\frac{d^2\bar{T}_1}{dx^2} + \beta^2\bar{T}_1 = 0, \quad \text{where} \quad \beta^2 = -\frac{s}{\alpha_1}$$

$$\frac{d^2\bar{T}_2}{dx^2} + \lambda^2\beta^2\bar{T}_2 = 0, \quad \lambda = \left(\frac{\alpha_1}{\alpha_2}\right)^{1/2}$$

$$\therefore \bar{T}_1 = A\sin\beta x + B\cos\beta x$$

$$\bar{T}_2 = D\sin\lambda\beta x + E\cos\lambda\beta x.$$

Using the Laplace transformed boundary conditions, solving for A, B, and inverting gives (from ref. [5])

$$T_1(x,t) = \frac{1}{2\pi i} \times \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{-Q\,e^{st}\,\mu\sin\lambda b\beta\sin\beta x + \cos\lambda b\beta\cos\beta x\,ds}{s\beta(k_2\sin a\beta\cos\lambda b\beta + \lambda k_2\cos a\beta\sin\lambda b\beta)}$$

where

$$\mu = \lambda\frac{k_2}{k_1}.$$

The singularities are given by the roots of

$$s\beta(k_1\sin a\beta\cos\lambda b\beta + \lambda k_2\cos a\beta\sin\lambda\beta b) = 0 \quad \text{(simple poles)}$$

and

$$s = 0 \quad \text{(double pole).}$$

Writing $p(s)$ as the numerator of the integral and $q(s)$ as the denominator, the residue at each of the simple poles β_n is given by $p(s_n)/q'(s_n)$ and the residue at $s = 0$ is given by refs. [5, 6] to be

$$\operatorname{Res}(0) = \frac{2}{3(q''(0))}[3p'(0)q''(0) - p(0)q'''(0)].$$

Hence

$$T_1(x,t) = \operatorname{Res}(0) - 2Q\sum_{n=1}^{\infty} e^{-\alpha_1\beta_n^2 t}\frac{(\mu\sin\lambda b\beta_n\sin x\beta_n + \cos\lambda b\beta_n\cos x\beta_n)}{\beta_n^2((ak_1+\lambda^2 bk_2)\cos a\beta_n\cos\lambda b\beta_n - \lambda(ak_2+bk_1)\sin a\beta_n\sin\lambda b\beta_n)}.$$

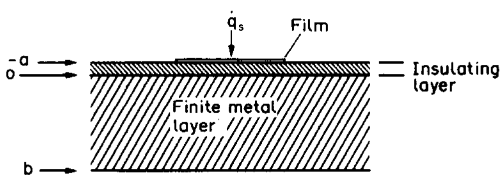

FIG. 4. Thin film gauge on two-layered composite comprising electrically insulating layer on finite metal layer.

The transcendental equation

$$k_1\sin a\beta\cos\lambda b\beta + \lambda k_2\cos a\beta\sin\lambda\beta b = 0$$

is solved to find the β_n using a standard NAG [7] library routine (CO5AGF) which locates a simple zero of a continuous function from a given starting value. An iterative procedure is adopted to find successive roots.

3.3. *Predicted surface temperature profiles*

The surface temperatures for two typical physical cases, quartz on nickel and Kapton on nickel, are shown in Figs. 5 and 6, respectively, for a step in heat transfer rate at the surface at $t = 0$, for the type 2 (two-layer semi-infinite) and type 3 (two-layer finite backwall) gauge solutions.

	Insulator	Metal substrate
Fig. 5(a)	200 μm quartz	3 mm nickel (finite backwall)
Fig. 5(b)	200 μm quartz	semi-infinite nickel
Fig. 6(a)	75 μm Kapton	3 mm nickel (finite backwall)
Fig. 6(b)	75 μm Kapton	semi-infinite nickel

The typical material properties used in these predictions are given in Table 1. The complete temperature–time–distance surface for each case is shown in Figs. 7 and 8, respectively. These are useful to predict the range of times for which a two-layered gauge with a finite backwall (type 3 gauge) can be considered to behave as a two-layered gauge with semi-infinite metal layer (type 2 gauge).

4. METHOD OF OBTAINING HEAT TRANSFER RATE FROM MEASURED SURFACE TEMPERATURE SIGNALS

The relationship between $\dot{q}_s$ and T_s for any system is

$$\bar{\dot{q}}_s = F(s)\bar{T}_s. \tag{6}$$

For the two-layered system (type 2 gauge), for example, from equation (3) putting $x = 0$

$$\bar{T}_s = \frac{\bar{\dot{q}}_s}{(s\rho_1 c_1 k_1)^{1/2}}\left(\frac{(1+A\exp\{-2a(s/\alpha_1)^{1/2}\})}{(1-A\exp\{-2a(s/\alpha_1)^{1/2}\})}\right).$$

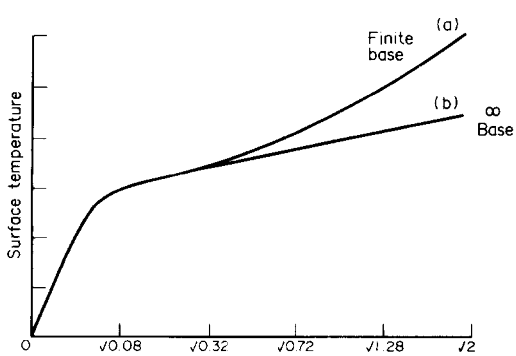

FIG. 5. Surface temperature signal from film subject to step in heat transfer rate at the surface: (a) 200 μm quartz on 3 mm nickel; (b) 200 μm quartz on semi-infinite nickel.

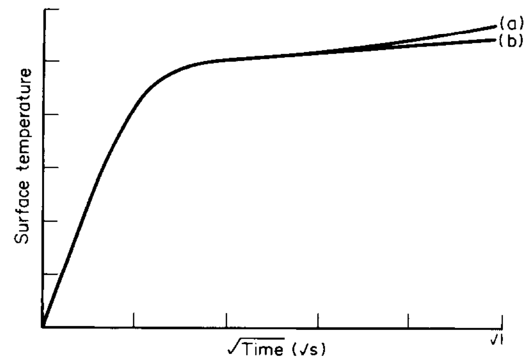

FIG. 6. Surface temperature signal from film subject to step in heat transfer rate at the surface: (a) 75 μm Kapton on 3 mm nickel; (b) 75 μm Kapton on semi-infinite nickel.

Hence

$$F(s) = (\rho_1 c_1 k_1 s)^{1/2}\frac{(1-A\exp\{-2a(s/\alpha_1)^{1/2}\})}{(1+A\exp\{-2a(s/\alpha_1)^{1/2}\})}.$$

To measure surface heat transfer rates, a constant current is passed through the thin film gauge, and the change of voltage, v caused by a change in surface temperature is prócessed by using an electrical analogue (Fig. 2).

The film voltage is given by

$$v = v_0 \alpha T \tag{7}$$

where α is the temperature coefficient of resistance of the film. The voltage output from the analogue is given by

$$\bar{v}_a = \frac{s^{1/2}}{k_a}\bar{v} = \frac{v_0}{k_a}s^{1/2}\alpha\bar{T}_s \tag{8}$$

where k_a is the analogue calibration constant (Fig. 2) assuming the analogue is ideal. Combining equations (6)–(8) gives

$$\bar{\dot{q}}_s = \frac{k_a}{v_0}\frac{F(s)}{\alpha s^{1/2}}\bar{v}_a. \tag{9}$$

For short times, $F(s) = (\rho_1 c_1 k_1 s)^{1/2}$ so

$$\bar{\dot{q}}_s = \frac{k_a \bar{v}_a}{v_0 \alpha}(\rho_1 c_1 k_1)^{1/2}. \tag{10}$$

If the analogue output is a step, then

$$\bar{v}_a = \frac{1}{s}$$

and the $\dot{q}_s$ to give this step would be

$$\bar{\dot{q}}_s = \frac{1}{\alpha}\frac{k_a}{v_0}F(s)s^{-3/2}. \tag{11}$$

For short times, from equation (10)

$$\bar{\dot{q}}_s = \frac{(\rho_1 c_1 k_1)^{1/2}k_a}{s v_0 \alpha}$$

so $\dot{q}_s^*(t)$ is a step of height $k_a(\rho_1 c_1 k_1)^{1/2}/v_0\alpha$ for small

Table 1. Material properties used in predictions

	Density (kg m^{-3})	Specific heat capacity (J kg^{-1} K^{-1})	Thermal conductivity (W m^{-1} K^{-1})	$\sqrt{(\rho ck)}$ (J m^{-2} s$^{-1/2}$ K^{-1})
Quartz	2200	755	1.425	1538
Kapton	1420	1090	0.155	490
Nickel	8900	450	84	18 342

time. Define the gauge step calibration function $h(t)$, with Laplace transform

$$H(s) = \frac{F(s)}{(\rho_1 c_1 k_1)^{1/2} s^{3/2}} \tag{12}$$

then from equations (12) and (11) for a unit step of v_a

$$\bar{q}_s(s) = \frac{1}{\alpha}\frac{k_a}{v_0} H(s)(\rho_1 c_1 k_1)^{1/2} \tag{13}$$

so

$$\dot{q}_s(t) = \frac{1}{\alpha}\frac{k_a}{v_0} h(t)(\rho_1 c_1 k_1)^{1/2}. \tag{14}$$

The sampled analogue output signal, with Laplace transform

$$\bar{v}_a = \frac{v_0}{k_a} s^{1/2} \bar{T}_s \alpha$$

can be considered to be a series of step functions such that

$$v_a(N\tau) = \sum_{n=1}^{N} a_n u(N\tau - n\tau) \tag{15}$$

where $u(t-\tau)$ is the delayed unit step function. Since $e^{-\tau s}F(s)$ is the Laplace transform of $f(t-\tau)$, where $F(s)$ is the Laplace transform of $f(t)$, the transform of equation (15) is

$$\bar{v}_a = \sum_{n=1}^{N} a_n \frac{e^{-st_n}}{s}$$

where

$$t_n = n\tau$$

and

$$a_n = v_a(n\tau) - v_a(n-1)\tau. \tag{16}$$

Then

$$\bar{q}_s(s) = \frac{1}{\alpha}\frac{k_a}{v_0} H(s)(\rho_1 c_1 k_1)^{1/2} \bar{v}_a(s) \tag{17}$$

so for $\bar{v}_a$ a sum of the series of step functions

$$\bar{q}_s(s) = \frac{1}{\alpha}\frac{k_a}{v_0} H(s)(\rho_1 c_1 k_1)^{1/2} \sum_{n=1}^{N} a_n e^{-st_n} \tag{18}$$

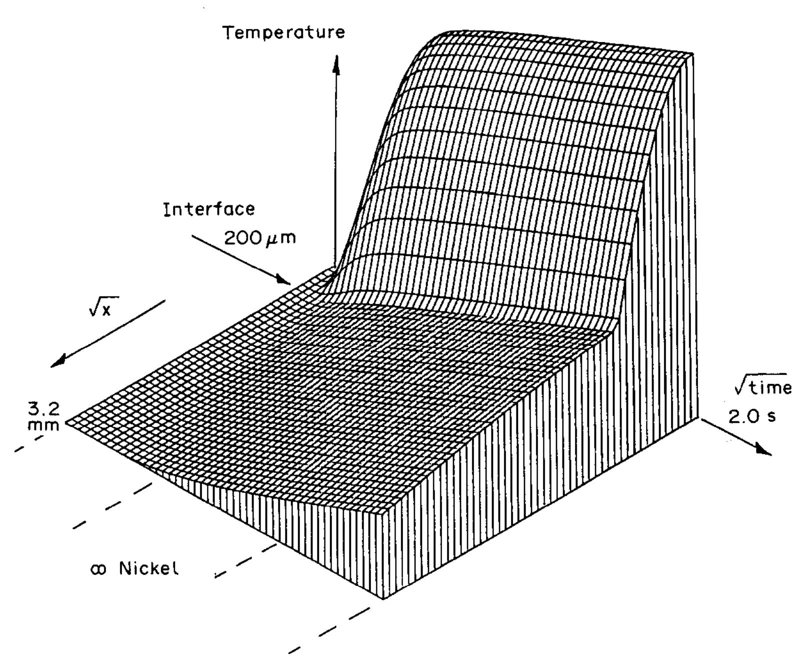

FIG. 7(a). Temperature–distance time surface for step in heat transfer rate at the surface of 200 μm quartz on semi-infinite nickel base.

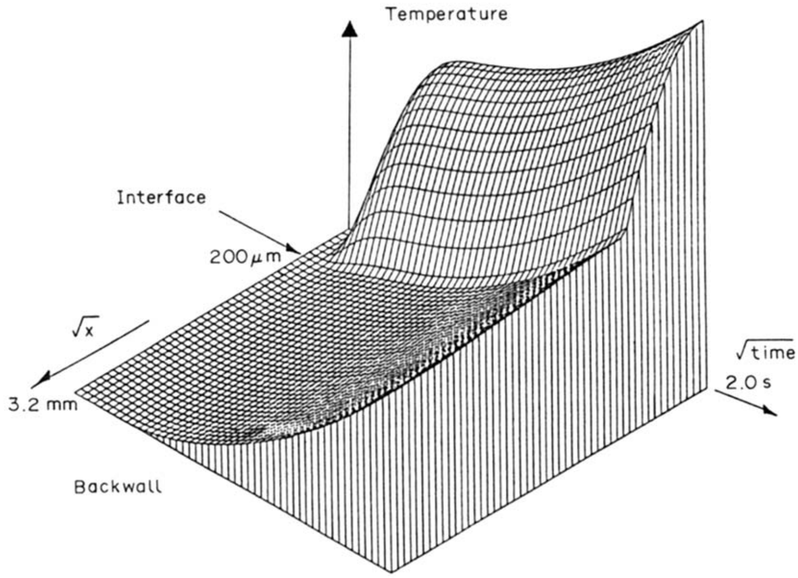

FIG. 7(b). Temperature–distance–time surface for step in heat transfer rate at the surface of 200 μm quartz on 3 mm nickel base.

thus inverting equation (18) gives

$$\dot{q}(N\tau) = \frac{1}{\alpha}\frac{k_a}{v_0}(\rho_1 c_1 k_1)^{1/2} \times \sum_{n=1}^{N} h(N-n)\tau(v_a(n\tau)-v_a(n-1)\tau) \quad (19)$$

so if $h(n\tau)$ is known at N discrete points, then $\dot{q}_s(N\tau)$, the sampled heat transfer signal can be computed. In practice, only about 200 points per channel of $v_a(n\tau)$, are sampled, so we only need to know $h(n\tau)$ at 200 points.

For one-layer (type 1), semi-infinite gauges, $h(t) = u(t)$, the unit step.

For two-layered, semi-infinite gauges (type 2

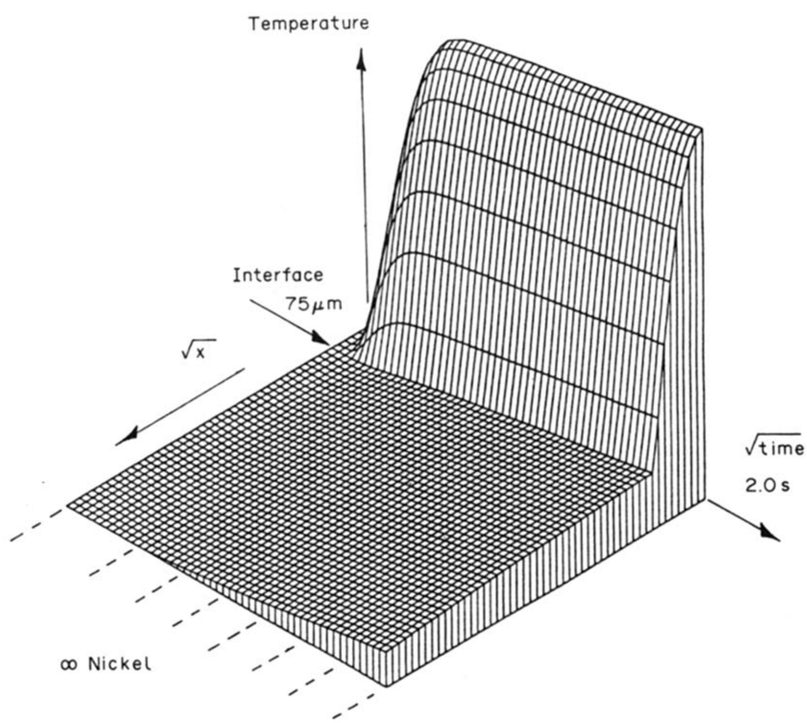

FIG. 8(a). Temperature–distance–time surface for step in heat transfer rate at the surface of 75 μm Kapton on semi-infinite nickel base.

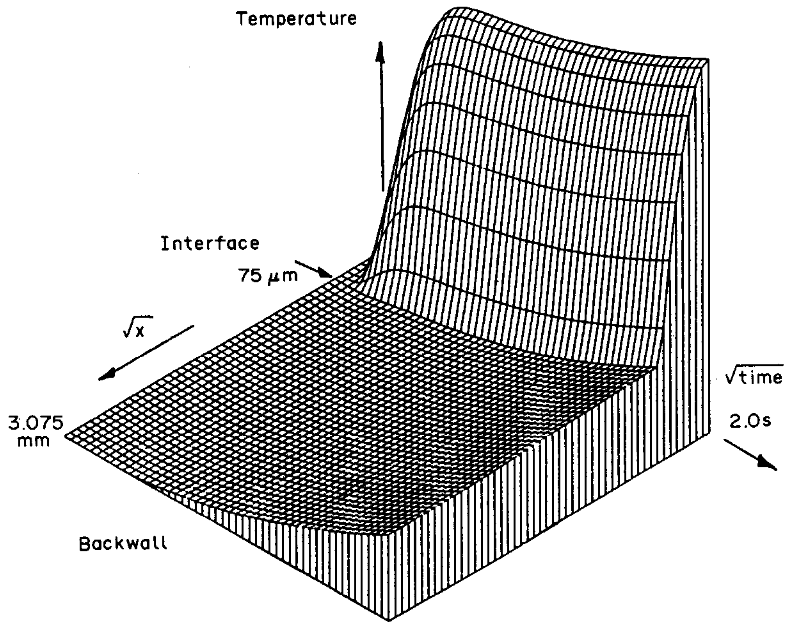

FIG. 8(b). Temperature–distance–time surface for step in heat transfer rate at the surface of 75 μm Kapton on 3 mm nickel base.

gauges) from equation (3)

$$F(s) = (\rho_1 c_1 k_1)^{1/2} s^{1/2} \frac{(1-A\exp\{-2a(s/\alpha_1)^{1/2}\})}{(1+A\exp\{-2a(s/\alpha_1)^{1/2}\})}.$$

Hence

$$H(s) = \frac{1}{s}\frac{(1-A\exp\{-2a(s/\alpha_1)^{1/2}\})}{(1+A\exp\{-2a(s/\alpha_1)^{1/2}\})} \quad (20)$$

so expanding the denominator gives

$$H(s) = \frac{1}{s}\left(1+2\sum_{m=1}^{\infty}(-1)^m A^m \times \exp(-2ma(s/\alpha_1)^{1/2})\right) \quad (21)$$

and inverting this gives

$$h(t) = 1+2\sum_{m=1}^{\infty}(-1)^m A^m \operatorname{erfc}(ma/(\alpha_1 t)^{1/2}). \quad (22)$$

For multi-layered (type 4) gauges $h(t)$ is not determined explicitly.

5. CALIBRATION

It has been shown from the above that it is necessary to determine $h(t)$ for a particular gauge in order to obtain the heat transfer rate. The following shows how this is possible for the various types of gauges.

5.1. *Two-layered gauge* (*type 2*)

It has been shown in equation (21) that $h(t)$ can be explicitly calculated if the values of $(\rho_1 c_1 k_1)^{1/2}$, $(\rho_2 c_2 k_2)^{1/2}$ and

$$a/(\alpha_1)^{1/2} = \frac{a}{k_1}(\rho_1 c_1 k_1)^{1/2}$$

are known.

For a step in heat transfer rate at the surface of a two-layer semi-infinite gauge

$$\bar{q} = \frac{Q}{s}.$$

From equation (5)

$$T_s = \frac{2Q}{(\rho_1 c_1 k_1)^{1/2}}\left((t/\pi)^{1/2} + 2\sum_{n=1}^{\infty} A^n \left\{(t/\pi)^{1/2} \times \exp\left(\frac{-n^2 a^2}{\alpha_1 t}\right) - \frac{na}{\alpha_1^{1/2}}\operatorname{erfc}(na/(\alpha_1 t)^{1/2})\right\}\right).$$

T_s can also be written as

$$T_s = 2Q\left(\frac{t}{\pi\rho_1 c_1 k_1}\right)^{1/2} + \frac{2Q}{(\rho_1 c_1 k_1)^{1/2}} \times \sum_{n=1}^{\infty} A^n (2t^{1/2} i\operatorname{erfc}(na/(\alpha_1 t)^{1/2})) \quad (23)$$

where

$$i\operatorname{erfc}(z) = \frac{2}{\pi}\int_z^{\infty}(t-z)\,e^{-t^2}\,dt.$$

For large t (from ref. [8])

$$i\operatorname{erfc}\left[\frac{na}{(\alpha_1 t)^{1/2}}\right] \simeq \frac{1}{\pi^{1/2}}\left(1+\frac{n^2 a^2}{(\alpha_1 t)}\right) - (na/(\alpha_1 t)^{1/2}).$$

So

$$T_s \simeq 2Q\left(\frac{t}{\rho_1 c_1 k_1 \pi}\right)^{1/2} + \frac{4Q}{(\rho_1 c_1 k_1)^{1/2}} \times \sum_{n=1}^{\infty} A^n \left\{\frac{n^2 a^2}{\alpha_1 (\pi t)^{1/2}} - (na/\alpha_1^{1/2}) + 2\left(\frac{t}{\pi}\right)^{1/2}\right\} \quad (24)$$

and

$$\sum_{n=1}^{\infty} nA^n = \frac{A}{(1-A)^2}.$$

Then since

$$\lim_{t\to\infty} \sum_{n=1}^{\infty} A^n (2n^2a^2/(\alpha_1(\pi t)^{1/2}) \to 0$$

for $t \to \infty$

$$T_s = 2Q\left(\frac{t}{\rho_2 c_2 k_2 \pi}\right)^{1/2} + \frac{aQ}{k_1}\left(1 - \frac{\rho_1 c_1 k_1}{\rho_2 c_2 k_2}\right). \quad (25)$$

This is effectively the metal alone with an offset to account for the presence of the insulator.

For short time, $t \to 0$, and $t^{1/2} \exp(-n^2a^2/\alpha_1 t) \to 0$ so

$$T_s(t) = 2Q\left(\frac{t}{\rho_1 c_1 k_1 \pi}\right)^{1/2} \quad (26)$$

which is the insulator alone.

If the two-layered substrate is subject to a step in heat transfer rate then, for a short time, the insulator is seen. Figure 9 shows the predicted surface temperatures for metal alone, insulator alone and insulator coated metal, plotted against $t^{1/2}$. The point of intersection of the straight lines in equations (25) and (26) is given by

$$t_1^{1/2} = \left(\frac{\pi}{2}\right)^{1/2} \frac{a}{k_1} \left(\frac{\rho_2 c_2 k_2}{\rho_1 c_1 k_1}\right)^{1/2} \times \{(\rho_1 c_1 k_1)^{1/2} + (\rho_2 c_2 k_2)^{1/2}\} \quad (27)$$

where $(\rho_1 c_1 k_1)^{1/2}$ is known from the standard air–glycerine test [1]. The ratio of the slope of the line given by equation (26) to that given by equation (25) is

$$\frac{\text{slope 1}}{\text{slope 2}} = \left(\frac{\rho_2 c_2 k_2}{\rho_1 c_1 k_1}\right)^{1/2}$$

hence $(\rho_2 c_2 k_2)^{1/2}$ is known. Then a/k_1 is calculated from the intersection point t_1 obtained by fitting straight lines for $t < t_1$ and $t > t_1$ on a T–$\sqrt{t}$ plot. As has been shown, this is the required constant in the equations for obtaining the calibration function $h(t)$.

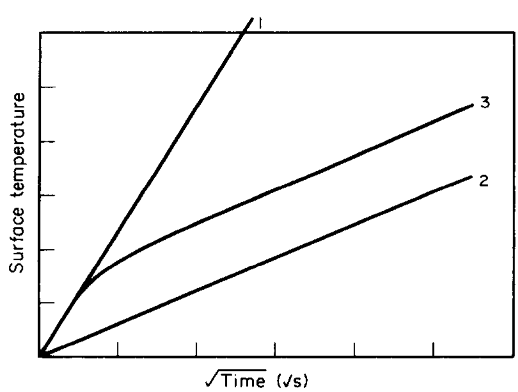

FIG. 9. Representative surface temperatures for step in heat transfer rate at the surface of: (1) insulator alone; (2) metal alone; (3) insulator coated metal.

5.2. *Multi-layered (type 4) gauges*

For a multi-layered (type 4) gauge, the following technique can be used to determine $h(t)$.

An air–glycerine test is used, for $t < t_1$, to determine $(\rho_1 c_1 k_1)^{1/2}$ for the top insulating layer. If a step in heat transfer rate is applied to the gauge, then the first part of the curve, for $t < t_1$, can be scaled, since

$$\bar{T}_s = \frac{\bar{\dot{q}}_s}{F(s)} = \frac{1}{sF(s)} \quad (28)$$

and for short times, $t < t_1$, $F(s) = (\rho_1 c_1 k_1 s)^{1/2}$ so

$$\bar{T}_s \simeq \frac{s^{-3/2}}{(\rho_1 c_1 k_1)^{1/2}}$$

or

$$T_s(t) = 2\left(\frac{t}{(\rho_1 c_1 k_1 \pi)}\right)^{1/2}.$$

Hence for $0 < t < t_1$, a parabola can be fitted through the measured curve and the result scaled.

Then from equation (12)

$$H(s) = \frac{F(s)s^{-3/2}}{(\rho_1 c_1 k_1)^{1/2}}$$

and from equation (28), for a unit step in $\dot{q}_s$

$$F(s) = \frac{1}{s\bar{T}_s} \quad (29)$$

so

$$H(s) = \frac{s^{-5/2}}{(\rho_1 c_1 k_1)^{1/2}\bar{T}_s}. \quad (30)$$

Thus

$$H(s)[(\rho_1 c_1 k_1 s)^{1/2}\bar{T}_s] = \frac{1}{s} \quad (31)$$

and putting

$$y(s) = (\rho_1 c_1 k_1 s)^{1/2}\bar{T}_s \quad (32)$$

gives

$$H(s)y(s) = \frac{1}{s^2}. \quad (33)$$

From the convolution theorem

$$\int_0^t h(u)y(u-t)\,\mathrm{d}u = t. \quad (34)$$

For $t < t_1$, $h(t) = 1$, hence

$$y(s) = (\rho_1 c_1 k_1 s)^{1/2} (\rho_1 c_1 k_1)^{-1/2} s^{-3/2} \tag{35}$$

so $y(t) = 1$ for $t \ll t_1$. Since $y(t)$ is known for a small time, numerical techniques can be used to compute $y(t)$ (by inverting equation (32)) without the usual difficulties. Alternatively an analogue can be used to compute $y(t)$ from the measured surface temperature.

The discrete form of equation (34) is

$$\sum_{n=1}^{N} h(n\tau) y(N-n)\tau = N\tau$$

or

$$h(N\tau) = N - \sum_{n=1}^{N-1} h(n\tau) y((N-n)\tau)$$

so $h(N\tau)$ can be computed step by step.

6. CONCLUSIONS

It has been shown that it is possible to use thin film gauges on two- or multi-layered substrates to obtain surface heat transfer measurements. Electrical analogues are still used as in the semi-infinite one-layer substrate case, and a heat transfer signal can be obtained by numerical methods from a sampled analogue output voltage signal. For high-speed heat transfer measurements, the usual single substrate semi-inifinite signal processing based on the insulating layer only can be used to determine the fluctuating components of $\dot{q}(t)$, over short timescales during which only the top layer plays a part in the heat conduction processes.

The practical implementation of this multi-layer gauge scheme has been demonstrated in ref. [9] and the gauges are now being used to measure heat transfer rates on metal nozzle guide vanes.

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LA THEORIE DES SONDES THERMIQUES PERFECTIONNEES A PLUSIEURS COUCHES MINCES

Résumé—On décrit la théorie d'une méthode transitoire de mesure des flux de chaleur sur des substrats métalliques recouverts par un isolant électrique, en utilisant des thermomètres à résistance en film mince. Ceci repose sur le système bien établi des substrats isolants semi-infinis. La nouvelle technique a une application dans les turbines car il y a actuellement un manque d'instrument convenable facile à construire et qui n'interfère pas avec l'écoulement. Le nouveau système décrit montre que l'on peut utiliser les sondes multicouches. On en fait l'analyse et on donne une estimation de dimensionnement et d'étalonnage.

DIE THEORIE VIELSCHICHTIGER DÜNNFILMSENSOREN BEI WÄRMEÜBERGANGSMESSUNGEN

Zusammenfassung—Dieser Aufsatz beschreibt die Theorie einer instationären Methode für die Messung des Wärmeübergangs an eine Metallschicht, welche mit einem elektrischen Isolator bedeckt ist. Dabei wird ein Dünnfilm-Widerstandsthermometer verwendet. Die Grundlage ist das bereits eingerichtete System, das halbunendliche Isolator-Unterschichten benutzt. Die Absicht ist, daß man diese Technik an den Testanlagen für Turbinen benutzen kann, denn es fehlt zur Zeit ein passender Meßfühler, der leicht angebaut werden kann und der die Strömung nicht stört. Das neue System, das hier geschildert wird, zeigt, daß man Vielschichtprüfköpfe benutzen kann. In diesem Aufsatz werden Analysen von Dünnschicht–Meßgeräten vorgenommen und Beschreibungen und Kalibrierungen einiger Muster mitgeteilt.

ТЕОРИЯ МНОГОСЛОЙНЫХ ТОНКОПЛЕНОЧНЫХ ДАТЧИКОВ ТЕПЛОПЕРЕНОСА

Аннотация—С использованием теории переходных процессов построен метод измерения интенсивности теплопереноса к металлическим подложкам, покрытым электроизолятором, в виде тонкопленочных термометров сопротивления. Рассматривается хорошо известная система, где применяется полуограниченная изолирующая подложка. Из-за отсутствия в настоящее время подходящих датчиков, простых в изготовлении и не оказывающих существенного воздействия на течение, предполагается новую методику использовать при проведении экспериментов в установках вращающихся турбин. Новая система подтвердила возможность использования датчиков с многослойной подложкой. Дан анализ сложных датчиков и приведены примеры расчетов и тарировки.